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A CONTINUOUS-GROUP CONCEPTION OF REDUNDANCY IN OBJECT IMAGES

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ABSTRACT. A method is described for constructing automatic algorithms. The method was tested for real materials using a sequence of frames of several motion picture films.

The problem of reducing redundancy in images is usually resolved on the /581* basis of a probability model in relation to the brightness distribution in the image plane. Another approach to the problem is founded on the fact that the images studied are object images, and to explain them one must consider the nature of object variation and how this variability occurs in the images. The model of an image or number of images of an object constructed on this basis is deterministic. In testing the ensuing algorithms for real image materials, we obtain results with respect to reduction of redundancy which significantly exceed those known previously.

Optical images are one of the most important means of storing and deliver- /582 ing information on certain objects or articles and the changes arising from them. The tremendous redundancy characteristic of this type of information can be explained, as we know, by the fact that the boundaries in the image between the object and the background or another object partially suppressed by it appear, strictly speaking, as the discontinuity of the function

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*Numbers in the margin indicate the pagination in the original foreign text.

describing the image. To reproduce with sufficient accuracy the appearance of the function at the discontinuity line (contrast), we must supply an image carrier with very high resolution (in television, on the order of $0.5 - 10^6$ units of resolution).

This resolution can be achieved only in locations with contrast and with fine details; in all more or less even portions of the image, it will amount to nothing (thereby increasing information content).

In relation to this property of images, attempts were made to free them from redundancy on the basis of a probability model which led to the conclusion that it was most probable for each image point to have neighboring points with very similar brightness and, less probably, to have highly dissimilar brightness. Thus, the problem is linked to the brightness decorrelation of the image. In sequences of moving images, we can likewise formulate the problem of using interimage connections to eliminate redundancy. As far as we know, the economy attained in this fashion generally does not exceed one-tenth of an order of /583
magnitude. This gain is slight compared to what one would expect intuitively, prompting a search for new principles to explain and remove redundancy — or, in other words, the creation of a new model, which expresses the essence of the redundancy characteristic of images.

Underlying the continuous-group approach to this problem is the shift from considering "images in general" as the subject of research to the consideration of <u>object images</u>. In this treatment, we shall deal, not with single images, but with a number of possible images of a given object.

In considering such a number of images, the question naturally arises: How do these images differ, or, what is it in them that varies?

If, for simplicity, we regard the conditions of surface illumination as constant, we may answer the question: Only a displacement of the object occurs from image to image (or center of projection) and perhaps, deformation of the object. But this means that there are only deformations and displacements, and for each of the images we must know whether one of them is "standard", to be remembered as a whole (also, however, which can be used as the above-mentioned probability of economy).

Thus, a model of an image or a number of images in the proposed concept can be interpreted in terms of variations arising from the object surface which is visible in the images. /584

To construct this model, we shall begin with a model of an object represented by an image. The first step in idealizing it is to consider the object as a three-dimensional body A in three-dimensional Euclidean space E, which is a model of the space surrounding us. As a method of projection, we consider the central projection (with a center at point $\bar{t}_o$ in space E) in a geometrical optics framework. Formally, this projection can be interpreted as the mapping of three-dimensional space E on two-dimensional space T. In constructing the model of an image in which we are interested, two basic properties of this mapping are utilized: continuity and unambiguity (of course, only on one side). These are the qualitative properties of the object used in the model.

It is assumed that the object A has an "external surface" $\mathcal{O}_A$, which is opaque. The portion of the external surface visible from point $\bar{t}_o$ is designated $\mathcal{O}_A(\bar{t}_o)$.

A hypothesis which is basic in this approach is that, for all possible changes in the object, its surface $\mathcal{O}_A$ either remains entirely equivalent to itself topologically or can be separated into a finite number of parts which remain topologically unchanged.

This property of the model is a formal expression of the hypothesized characteristic of real objects made of connected particles which do not scatter in an arbitrary manner. It is assumed that the "topological stability" indicated is a very widely distributed property in all its specifics. Thus, /585 it is satisfied by any hard bodies, elastic bodies (such as living materials), typographical or handwritten symbols, etc.[1].

A formal statement of the properties enumerated above and some additional ones subject to consideration in the image model are given in [1] in the form of a I - UP* restriction.

Let us now consider how the properties of object model and the projection method described above occur in the image model. We will need to introduce the following notation:

$\bar{t}$ — point on object surface;
$t=(t_1,t_2)$ — point in plane T at which point $\bar{t}$ is represented (projected); t_1 and t_2 are its coordinates in the plane;
$x(t)$ — function describing the brightness of the image at all points, referred to hereafter simply as the image.

Now let x(t) and x'(t) be two images of object A for which the visible regions of its surface $\theta_A(\bar{t}_0)$ and $\theta_A'(\bar{t}_0')$ have a portion in common. /586

Let the projection of this common portion occupy a part of the area of image x(t) designated as T_A and a part of image x'(t) designated as T_A'.

From the models of the object and the method of projection considered above, we may now deduce the following properties of the image model which are central in this approach:

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[1] For hard bodies and typographical symbols, the topological invariability degenerates frequently into geometrical invariability.

*Translator's Note: This may designate I-universal device.

τ_t and τ_t' are regions of the plane such that either of them is entirely topologically equivalent to the other, and they may be broken down into an equal finite number of pairs of topologically equivalent parts.

In other words, a part of image x(t) occupying region τ_t may be converted to the corresponding part of image x'(t) occupying region τ_t' by topological (i.e. not by breaking the plane or squeezing it into folds) transformation of either the entire plane occupied by region τ_t, or each of its several portions.

Let us note for later reference that, if point $\bar{t}$ belongs to the common visible portion of the surface, then its forms t and t' in images x and x' are called corresponding points of the images; obviously $t \in \tau_t$ and $t' \in \tau_t'$.

Thus, since the aggregate of material particles is an "object" and may be identified as an "object" only when these particles are not displaced in space independently and arbitrarily (do not "scatter") but are linked to one another, at least within the limits of the definite proximity, in a number of object images there necessarily ensue strict correlations between them. These correlations are expressed by the possibility of converting one of two object images (or parts) into the other (in the presence of a common visible portion of the surface) through topological transformation of the corresponding regions of the plane. It is hypothesized that these correlations obey a very definite pattern which must be utilized to eliminate redundancy. /587

To construct the corresponding working apparatus, one must consider the problem of the means of describing (approximation) the transformations and the quantities involved.

From the model it follows [1] that the adequate analytical expression of the approximation will be an equation for a continuous group of transformations (Lie group) of the plane.

The general form of these equations is:

$$t_1'^* = f_1(t_1, t_2; a_1, \ldots, a_r)$$
$$t_2'^* = f_2(t_1, t_2; a_1, \ldots, a_r) \qquad (1)$$

where t_1, t_2 are the old, and $t_1'^*, t_2'^*$ are the new (after transformation) running coordinates of the image points; $a_1, \ldots, a_r$ are the transformation parameters; r is generally small. /588

The quality of the approximations will be good if for each point $t \in T_x$ of image x, the calculated position $t'^* = (t_1'^*, t_2'^*)$ corresponding to a point in image x' differs from its true position $t' = (t_1', t_2')$ by no more than a certain tolerable amount.

The parameters $a_1, \ldots, a_r$ in the Lie group equations are <u>essential</u> [2], meaning that with a smaller quantity of numbers it is impossible to describe the transformation of a given section of the plane for a given approximation error. Thus, the use of the equations described implies the possibility of describing the regularity between images in the <u>most theoretical economical fashion</u>. The selection algorithm for Equations (1) and for calculating the parameters $a_1 - a_r$ for the approximation of a specific transformation is given in [1].

Thus, in the presence of one or several standard images, any other image from a number of images of a given object can be fully given by a set of r numbers.

Especially apparent economy occurs in the case when the object images are ordered in <u>sequential</u> form, as in a strip of motion picture film /589
related to a single subject.

In this case, the sequence of values from frame to frame of a certain parameter a_i forms a sequence of calculations for a continuous function $a_i(\nu)$

(where ν designates time).

The obtained τ functions of $a_j(\nu)$ may be interpolated by taking, in order, readings considerably less frequently than the frame frequency. It can be shown that with the proper selection of approximating equations and transforming portions of the plane (in piece-by-piece approximation of the transformations) one can obtain the theoretically maximum economy in the group of numbers describing the separate image and the entire sequence, which means that the theoretically least redundancy is retained.

On the basis of the method described, automatic algorithms were constructed through which the method was tested for real materials using a sequence of frames from strips of several motion picture films [3] (average length of sequence with a single subject — $\approx$ 100 frames). The reduction of redundancy was measured by the relationship of the number of figures describing the sequence of images in ordinary element-by-element transmission to the number of figures involved in the sequence using the method described here. The /590 experiments yielded the following results [4]. Aside from the need to transmit a standard frame, the total gain amounted to a factor of 300. Taking into account the standard image, the gain was equal to 76; even in the case where point corrections were necessary, which could be required at the junctions of parts transformed piece-by-piece, the gain is on the order of 70. Synthetic images were indistinguishable from natural ones.

REFERENCES

1. Fayn, V. S. Image Identification (Continuous-Group Theory). Collection: Avtomaticheskoye chteniye teksta (Automatic Text Reading). (VINITI), "Nauka" Press, Moscow, 1967.

2. Chebotarev, N. G. Teoriya grupp Li (Lie Group Theory). GITTL, Moscow, Leningrad, 1940.

3. Fayn, V. S. and V. S. Vaynshteyn. On One Approach to the Transmission of Moving Images. Tezisy dokladov na II Vsyesoyuznoy konferentsii po primeneniyu televizionnovychislitel'noy tekhniki v proizvodstve (Reports of II All-Union Conference on Application of Television-Computer Techniques in Industry), Moscow, 1966.

4. Fayn, V. S., V. N. Sorokin and V. S. Vaynshteyn. Continuous-Group Identification of Forms, Some Experimental Results. Izvestiya AN SSSR, Tekhnicheskaya kibernetika, No. 6, 1969.

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